Mark Scheme (Results)

Summer 2018

Pearson Edexcel GCE Mathematics
Decision D1 Paper 6689_01

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General Marking Guidance

• All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.

• Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.

• Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.

• There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.

• All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.

• Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.

• Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.
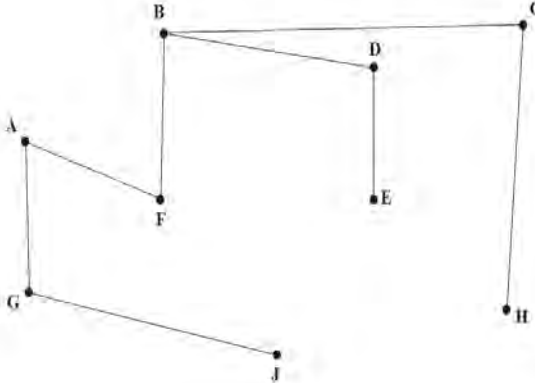
EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

*   **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
*   **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
*   **B** marks are unconditional accuracy marks (independent of M marks)
*   Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

*   bod – benefit of doubt
*   ft – follow through
*   the symbol $\sqrt{}$ will be used for correct ft
*   cao – correct answer only
*   cso - correct solution only. There must be no errors in this part of the question to obtain this mark
*   isw – ignore subsequent working
*   awrt – answers which round to
*   SC: special case
*   oe – or equivalent (and appropriate)
*   dep – dependent
*   indep – independent
*   dp decimal places
*   sf significant figures
*   ✱ The answer is printed on the paper
*   ⌐ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    *   If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    *   If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

Question Number	Scheme	Marks
1. (a)(i)	A tree is a connected graph with no cycles	B1
(a)(ii)	A minimum spanning tree is a tree that contains all vertices and the total length of its arcs is as small as possible	B1 B1 **(3)**
(b)	AG, AF, GJ; FB, BC, BD; CH, DE	M1; A1; A1 **(3)**
(c)		B1
	(Weight of the tree is) 182	B1 **(2)**
		8 marks

Notes for Question 1

In (a) all technical language used must be correct

ai1B1: Connected + **no cycle(s)** (must contain these two points – do not allow 'circle', 'loop' etc. for cycle(s)) – for connected allow 'a graph in which a path exists between each pair of vertices'
aii2B1: Contains **all (oe)** and either **vertices** or **nodes**
aii3B1: Total length (of arcs) is **minimised** (must contain the two points regarding total and minimised/smallest but need not necessarily mention arcs e.g. 'smallest total weight' is sufficient for this mark). However 'a MST is a tree that contains all the nodes and has the least possible weight' would score B1B0 in (a)(ii) as they have 'all the nodes', but no explicit mention of 'total' (or equivalent e.g. 'sum of')

b1M1: First three arcs (AG, AF, GJ) correctly chosen, or first four nodes (A, G, F, J) correctly chosen in order. **If any explicit rejections seen then M1 (max) only**. A list of weights only scores M0. Candidates may apply Prim's in matrix form so the order of the nodes may be seen at the top of a matrix – accept {1,-,-,-,-,3,2,-,4} for the M mark. Allow GJ for JG etc. throughout (b)
b1A1: First six arcs correctly chosen in order (AG, AF, GJ, FB, BC, BD), **or** all nodes correctly chosen in order (A, G, F, J, B, C, D, H, E). Candidates may apply Prim's in matrix form so the order of the nodes may be seen at the top of a matrix – accept {1,5,6,7,9,3,2,8,4} – do not condone any missing numbers e.g. the number 9 must be above E
b2A1: CSO (correct solution only) – all arcs correctly stated and chosen in the correct order. Candidates must be considering arcs for this final mark (do not accept a list of nodes or numbers across the top of the matrix unless the correct list of arcs (in the correct order) is also seen)

Misread: Starting at a node other than A scores M1 only in (b) – must have the first three arcs (or four nodes) correct (and in the correct order)

c1B1: CAO (tree)
c2B1: CAO (182)

Question Number	Scheme	Marks	
2. (a)	(If starting at the left hand end of the list then) in the first pass we compare the first value with the second value and we swap these values if the second is larger than the first. We then compare the value which is now second with the third value and swap if the third is larger than the second. We continue in this way until we reach the end of the list	M1 A1	**(2)**
(b)	If sorting from left to right then the smallest number (15) would be in the correct position	B1	
	If sorting from right to left then the largest number (35) would be in the correct position	B1	**(2)**
(c)	27	B1	**(1)**
(d)	30 33 35 27 20 24 21 15 19 35 33 30 27 24 21 20 15 19 35 33 30 27 24 21 20 19 15 35 33 30 27 24 21 20 19 15 (sort complete) **SEE SC BELOW FOR THOSE USING 20 AS A PIVOT**	M1 A1 A1	**(3)**
(e)	Bin 1: 35 24 Bin 2: 33 27 Bin 3: 30 21 Bin 4: 20 19 15	M1 A1 A1	**(3)**
(f)	$\dfrac{242}{60} = 4.0333...$ so no it is not possible to pack the 10 numbers into 4 bins of size 60 **or** a wastage argument regarding the maximum possible value of 16	B1	**(1)**
		12 marks	

Notes for Question 2

a1M1: Comparing first value with second value, swap if second is larger (oe) – must be clear that the first value in the list is being compared with the second value in the list and swapping if the second is larger than the first (oe)

a1A1: Compare second with third, (and then third with fourth), and so on until the end (or 9th item) of the list – must be clear that after the first comparison the second value in the list is compared with the third value and so on until the end of the list

b1B1: CAO (for left to right) – must mention either the smallest (oe) or the 15 should be at the end of the list/correct position (bod if 15 is mentioned (but not incorrectly))
b2B1: CAO (for right to left) – must mention either the largest (oe) or the 35 should be at the end of the list/correct position (bod if 35 is mentioned (but not incorrectly))

c1B1: CAO (27) – B0 if choice of answers

d1M1: Quick sort – 33 and 21 selected as pivots and after their first pass the sublists must read (values greater than the pivot), pivot, (values less than the pivot)
d1A1: The first 'two' passes correct – they do not need to be selecting a pivot for the 'third' pass
d2A1: CSO (correct solution only) – must include a 'third' pass in which the 19 (or 20 if middle left) is used as a pivot – a 'sort complete' statement is not required (as their third pass contains no swaps)

SC for (d): If candidates start the quick sort on the list given in the stem to (b) then award M1 only for 20 chosen as a pivot and after the first pass the list must read 30 33 35 27 24 21 20 15 19
e1M1: 35 in Bin 1, Bin 2 correct and 30 in Bin 3 (so first 4 values correctly placed) – no follow through on an incorrect list from (d) – condone cumulative totals for M1 only (the boxed values)
e1A1: First 7 values correctly placed (the boxed and underlined values)
e2A1: CSO (so no additional/repeated values)

f1B1: CSO – therefore correct calculation (e.g. 242/4 = 60.5) + conclusion (oe i.e. an argument based on wastage e.g. there is only room for a tenth value of at most 16)

In (d) sorting list into ascending order is M0 but allow recovery in (e) if first-fit decreasing

Middle left for (d):

30	<u>33</u>	35	27	20	24	<u>21</u>	15	19
35	33	30	27	24	21	20	<u>15</u>	19
35	33	30	27	24	21	<u>20</u>	19	15
35	33	30	27	24	21	20	19	15

Question Number	Scheme	Marks
3. (a)		M1 (7 activities, 1 start + 2 dummies) A1 (ABCD+1st 2 dummies) A1 (EFGH) A1 (IJK+3 dummy) A1 (CSO) **(5)**
(b)	Given that D is critical this implies that A and E are guaranteed to be critical	M1 A1 **(2)**
		7 marks

<div align="center">Notes for Question 3</div>

Condone lack of, or incorrect, numbered events throughout. 'Dealt with correctly' means that the activity starts from the correct event but need not necessarily finish at the correct event, e.g. 'G dealt with correctly' requires the correct precedences for this activity, i.e. A, B and C labelled correctly and leading into the same node and G starting from that node but do not consider the end event for G. **Activity on node is M0. Note that additional/unnecessary dummies that do not break the precedence condition can earn the first three A marks but will lose the final A mark (the CSO mark). However, additional unlabelled activities will lose the corresponding A marks if they effect the 'dealt with correctly' condition for other activities**

If an activity, say C is not labelled (but the arc is present) then this will lose the first A mark and the final (CSO) A mark – they can still earn the second and third A marks on the bod.

Ignore lack of arrows on the activities for the first four marks only. If no arrows on any dummies then maximum mark in (a) is M1 only

a1M1: Seven activities (labelled on arc), one start and at least two dummies placed

a1A1: Activities A, B, C, 1st two dummies (including arrows on these two dummies) and D dealt with correctly. The first two dummies are those at the end of activities A and B

a2A1: Activities E, F, G and H dealt with correctly

a3A1: 3rd dummy (including arrow on this dummy at the end of activity F) and activities I, J and K dealt with correctly

a4A1: CSO (all previous marks must have been awarded) – final dummy correctly placed (+arrow), **all** arrows present, exactly four dummies correctly placed with one finish. **Please check all arcs carefully for arrows**
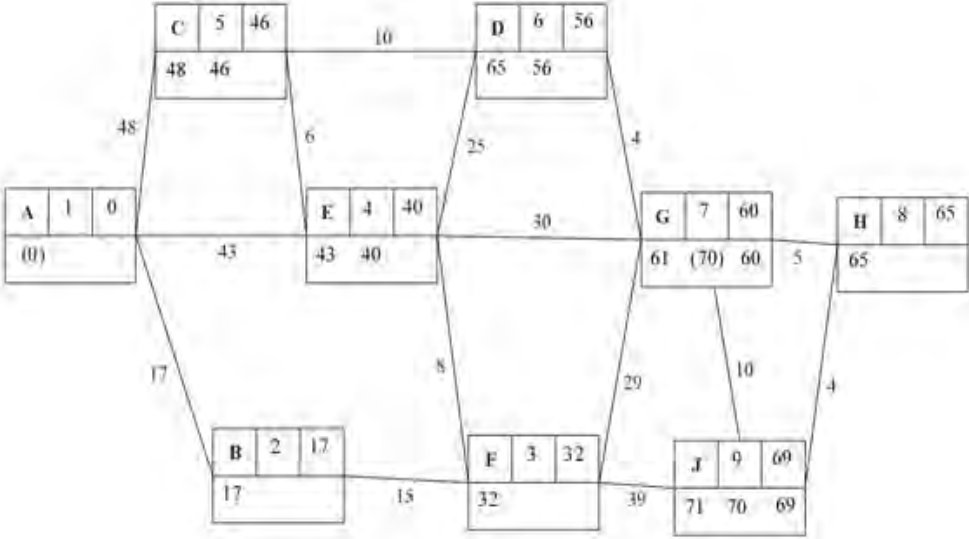
Note additional valid solutions:
- the arrow on the final dummy between I and J reversed so that activity I will now end at the finish node
- Activities I and J interchanged
- A combination of both points above (i.e. I and J interchanged and the arrow on the dummy reversed)

Therefore it is vital that the diagram is checked carefully for these other equally acceptable/valid solutions

b1M1: One correct activity with at most 4 activities stated (ignore any mention of D in this part)

b1A1: Both (A and E) correct and no others

Question Number	Scheme	Marks
4. (a)	Shortest time: 69 (minutes) Quickest route: ABFECDGHJ	M1 A1 (ABFE) A1 (CDG) A1ft (HJ) A1ft A1 **(6)**
(b)	A(BFE)C + D(C)E = 46+ 16 = 62 A(BFEC)D + CE = 56 + 6 = 62 A(BF)E + CD = 40 + 10 = 50* Repeat edges: AB, BF, EF and CD Duration = 293 – 10 + 50 = 333 (minutes)	M1 A1 A1 A1 A1 A1 **(6)**
(c)	Vertex E: 3 Vertex F: 3	B1 B1 **(2)**
		14 marks

Notes for Question 4

In (a) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at J the working values must be 71 70 69 in that order (so 70 71 69 is incorrect)
It is also important that the order of labelling is checked carefully – some candidates start with a label of 0 at A (rather than 1) – which is fine. Also the order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, … will be penalised once (see notes below) but 1, 2, 3, 5, 6, … is fine. Errors in the final values and working values are penalised before errors in the order of labelling

a1M1: A larger value replaced by a smaller value at least once in the working values at either C or D or E or G or J
a1A1: All values in A, B, F and E correct and the working values in the correct order at E (including order of labelling). Condone lack of 0 in A's working value
a2A1: All values C, D and G correct and the working values in the correct order. Penalise order of labelling only once per question (C, D and G must be labelled in that order and C must be labelled after A, B, F and E). Note that an additional working value of 70 at G betweeen the 61 and 60 is not an error so 61 70 60 is fine, however, any other number or the 70 not in this position is incorrect and scores A0

a3A1ft: All values in H and J correct on the follow through and the working values in the correct order. Penalise order of labelling only once per question. To follow through H check that the working value at H follows from the candidate's final values from node G and that the final value, and order of labelling, follows through correctly. Repeat this process for J (which will have working values from F, G and H with the order of these values determined by the candidates order of labelling of F, G and H)
a4A1ft: Their final value at J **only** (condone lack of units)
a5A1: CAO - correct route (from either A to J or J to A)

b1M1: Three distinct pairings of A, C, D and E
b1A1: **Any** row correct including pairing **and** total
b2A1: **Any** two rows correct including pairings **and** totals
b3A1: All three rows correct including pairings **and** totals
b4A1: CAO correct edges clearly stated (and not just in their working) as AB, BF, EF, CD. Do not accept AE, ABFE or AE via B and F
b5A1: CAO (333)

c1B1: CAO (vertex E)
c2B1: CAO (vertex F)

In (c) if answer lines not used then:

3, 3 – B1 B1
3 + 3 = 6 – B1 B1
3, x where $x \neq 3$ – B1 B0
x, 3 where $x \neq 3$ – B0 B1
3 – B1 B1
6 – B0 B0

If after (b) a 3 appears only and it is not labelled as (c) then award no marks

Question Number	Scheme	Marks	
5. (a)	Initial matching: (C unmatched), D = 4, H = 1, (R unmatched), S = 3	B1	**(1)**
(b)	Alternating path: C – 1 = H – 3 = S – 4 = D – 5	B1	**(1)**
(c)	Improved matching: C = 3, D = 5, H = 1, (R unmatched), S = 4	B1	**(1)**
(d)	Alternating path: R – 4 = S – 3 = C – 1 = H – 2	M1	
	Change status to give: R = 4 – S = 3 – C = 1 – H = 2	A1	
	Complete matching: C = 1, D = 5, H = 2, R = 4, S = 3	A1	**(3)**
		6 marks	
Notes for Question 5			

a1B1: CAO (D = 4, H = 1, S = 3) – accept on a **clear** diagram (with three arcs only)

b1B1: CAO (alternating path) – allow any unambiguous notation e.g. C – 1 – H – 3 – S – 4 – D – 5 but do not allow C – 1, H – 3, S – 4, D – 5 (as this is not a path but a matching)

c1B1: CAO (C = 3, D = 5, H = 1, S = 4) – accept on a **clear** diagram (with four arcs only)

d1M1: An alternating path (e.g. letter 1st set – number 2nd set – letter 1st set – …) from R to 2 (or vice-versa)
d1A1: CAO – a correct path including change status **either** stated (only accept 'change (of) status' **or** 'c.s' but not, e.g.'change state') **or** shown (all symbols e.g. (…– … = … – …) interchanged (… = …. – … = …))
Chosen path clear
d2A1: CAO – complete matching - must follow from the correct stated path. Accept either stated **or** on a **clear** diagram (with five arcs **only**)

Question Number	Scheme	Marks
6. (a)		M1 A1 M1 A1 **(4)**
(b)	Critical activities: A, D, J and N	B1 **(1)**
(c)		M1 A1 A1 A1 **(4)**
(d)	Minimum workers is 4 e.g. D, E, F and G together with $11 <$ time < 12	M1 A1 **(2)**
		11 marks

Notes for Question 6

a1M1: All top boxes complete, values in the top boxes generally increasing in the direction of the arrows ('left to right'), condone one 'rogue' value (if values do not increase in the direction of the arrows then if one value is ignored and then the values do increase in the direction of the arrows then this is considered to be only one rogue value)
a1A1: CAO for the top boxes
a2M1: All bottom boxes complete, values generally decreasing in the opposite direction of the arrows ('right to left'), condone one rogue. Condone missing 0 and/or 35 for the M only
a2A1: CAO for the bottom boxes

b1B1: CAO (A, D, J and N)

c1M1: At least twelve activities including at least six floats. A scheduling diagram only scores M0 – however, if a scheduling diagram appears after a Gantt chart then mark Gantt chart and isw their scheduling
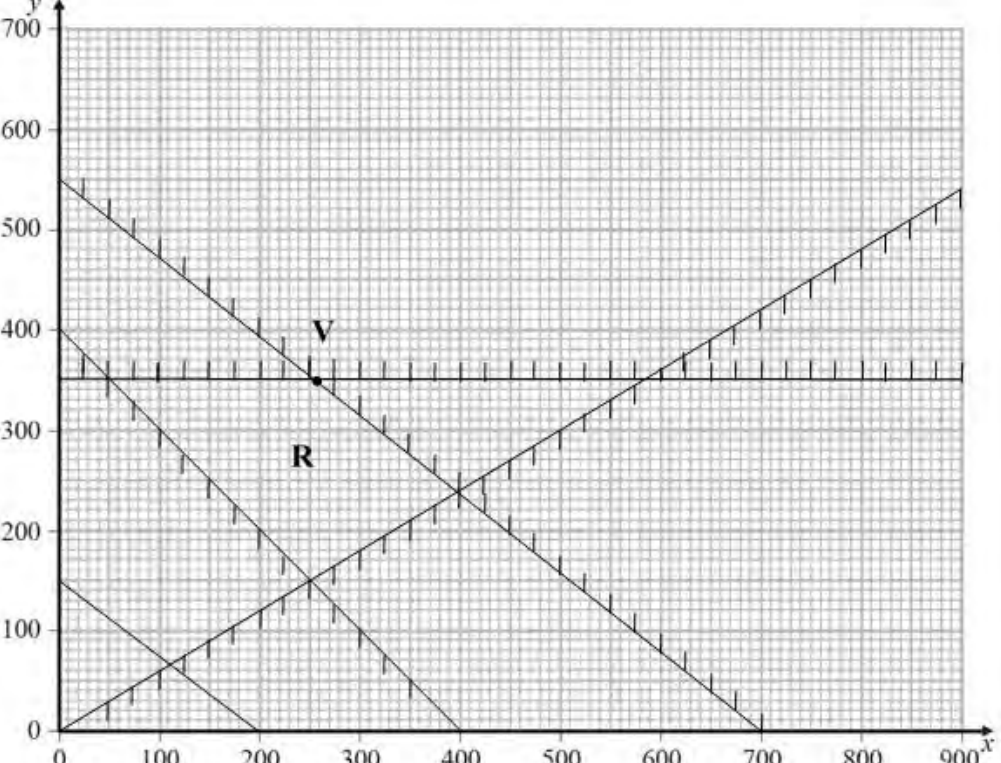**c1A1:** The critical activities dealt with correctly and appearing just once (A, D, J and N) and three non-critical activities dealt with correctly
c2A1: Any seven non-critical activities correct (this mark is not dependent on the previous A mark)
c3A1: CSO – completely correct Gantt chart (exactly fourteen activities appearing just once)

d1M1: Either a statement with the correct number of workers (4) and the correct activities (D, E, F and G) with any numerical time stated **or** the correct number of workers (4) and a correct time
d1A1: A completely correct statement with details of both time and activities. Candidates only need to give a time within the correct interval of $11 < \text{time} < 12$. Please note the strict inequalities for the time interval (e.g. implying a time of 11 is incorrect). Answers given as an interval of time are acceptable provided the time interval stated is correct for all its possible values (e.g. 'time $11 – 12$' is A0, 'in the interval $11 – 12$' is A0 but 'betweeen 11 and 12' is A1). Allow for example, 'on day 12' as equivalent to $11 < \text{time} < 12$

Question Number	Scheme	Marks
7. (a)	Maximise $0.75x + y$	B1
	Subject to $x + y \geq 400$	B1
	$y \leq 350$	B1
	$5y \geq 3x$	M1 A1
	$11x + 14y \leq 7700$	B1 (6)
(b)		B1 B1 B1 B1 (4)
(c)	Drawing an objective line accept reciprocal gradient	M1
	Correct objective line	A1
	V correctly labelled	A1 (3)
(d)	$V\left(\dfrac{2800}{11}, 350\right)$	M1 A1 (2)
(e)	(The manager should buy) 254 plain (scones) and 350 fruit (scones)	B1
	Profit is (£) 540.50	B1 (2)
		17 marks

Notes for Question 7

a1B1: Expression correct (or $75x + 100y$) <u>together</u> with 'maximise' or 'max' but not 'maximum' – isw if coefficients are subsequently simpified but either $75x + 100y$ or $0.75x + y$ must be seen at some point for this mark to be awarded. The 'max' must appear beside or suitably close to one of the correct two expressions

a2B1: CAO ($x + y \geq 400$)

a3B1: CAO ($y \leq 350$)

a1M1: $5y \square 3x$ where $\square$ is any inequality or equals. Accept $3y \geq 5x$ for this mark. **M0 if coefficients are not integers**

a1A1: CAO ($5y \geq 3x$)

a4B1: CAO ($11x + 14y \leq 7700$)

In (b), lines must be long enough to define the correct feasible region and pass through one small square of the points stated:

$x + y = 400$ must pass within one small square of its intersection with the axes – (0, 400) and (400, 0)

$11x + 14y = 7700$ must pass within one small square of its intersection with the axes – (0, 550) and (700, 0)

$5y = 3x$ must pass within one small square of (0, 0) and if extended pass through (500, 300)

$y = 350$ must pass within one small square of (0, 350) and if extended pass through (500, 350)

b1B1: Any two lines correctly drawn

b2B1: Any three lines correctly drawn

b3B1: All four lines correctly drawn

b4B1: Region, R, correctly labelled – dependent on scoring the first three marks in this part

c1M1: Drawing their objective line (based on their answer to (a)) or its reciprocal – if their line on the graph is shorter than the length equivalent to that of the line from (0, 37.5) to (50, 0) then M0. Line must be correct to within one small square if extended from axis to axis. **Their line must have a negative gradient**

c1A1: Drawing the correct objective line – same condition that the line must be correct to within one small square if extended from axis to axis

c2A1: The correct V labelled or clearly identified on their graph – **note that this mark is dependent on scoring at least B1B1B1B0 in (b) <u>and</u> the two previous marks in this part**

d1M1: Must have scored at least B1B1B0B0 in (b) <u>and</u> candidates must have drawn an objective line (but note that it does not need to be correct but must have negative gradient). Must be solving one of the following three pairs of equations **only**: $11x + 14y = 7700, y = 350$ **or** $11x + 14y = 7700, 5y = 3x$ **or** $5x = 3y, y = 350$. Must be a correct method to solve simultaneous equations and must arrive at $x = \dots$ and $y = \dots$ but allow slips/errors. This mark can also be awarded for the correct exact coordinates stated with no working provided B1B1B0B0 in (b) and an objective line drawn

d1A1: Correct exact coordinates of V either derived or stated (so no working required) as either $\left(\dfrac{2800}{11}, 350\right)$ or $\left(254\dfrac{6}{11}, 350\right)$. **Note that this mark is dependent on B1B1B1B0 scored in (b) <u>and</u> a correct objective line**

e1B1: CAO in context – so not in terms of x and y only – **dependent on B1B1B1B0 in (b) <u>and</u> a correct objective line**

e2B1: CAO (allow 540.5) – **dependent on B1B1B1B0 in (b) <u>and</u> a correct objective line** – condone lack of units if given in £ (or 54050p – if given in pence, however, units must be given)